Tensile Behavior of Symmetrical Laminates Made of PCBA and PMMA Layers

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ABSTRACT: The stiffness and strength of a composite material in form of laminate is obtained from the properties of the constituent laminae. The stacking sequence of the laminate affects the mechanical behavior. The interface between different laminae is also an important factor, since it influences the stresses that are developed in the laminate, and hence the strengths. In this study, a theoretical investigation of the mechanical behavior of symmetrical laminates made of isotropic layers [Lexan (PCBA) and Plexiglas (PMMA)] was attempted. An analysis based on the lamination theory was performed to determine the stress distribution and strains as well as the elastic constants. Experimental measurements with specimens made of laminates with different stacking sequences were carried out. The obtained values were compared with the theoretical values given by the lamination theory and mechanics of materials approach. © 2006 Wiley Periodicals, Inc. J Appl Polym Sci 101: 3215–3226, 2006

Key words: laminate; stacking sequence; interface; strength; elastic constants

INTRODUCTION

A laminate is a stack of laminae of isotropic or nonisotropic materials. In recent years, laminated composite materials present great interest especially for lightweight constructions demanding high strength. Laminated composites consist of layers of at least two different materials that are bonded together. Lamination is used to combine the best aspects of the constituent layers to achieve a more useful material. The properties that can be emphasized by lamination are strength, stiffness, low weight, corrosion resistance, wear resistance, thermal insulation, acoustical insulation, etc. A major purpose of lamination is to tailor the directional dependence of strength and stiffness of a material to match the loading environment of the structural element. Laminates are uniquely suited to the this objective, since when they are made of layers of unidirectional fibers, the principal material directions of each layer can be obtained according to the need. They are also made of isotropic layers and are usually known as sandwich materials. Laminated composites have been considered theoretically by many investigators. In ref. 1, a theoretical solution was proposed for the case of a multilayered laminated composite beam under end load. Ref. 2 represents a more general solution than the previous one. Ref. 3

presents a plane stress solution applicable to a thin-walled cantilever beam with end load. Ref. 4 extended the preceding result to include the influence of beam width for the Saint-Venant solution to the bending of a sandwich beam. Lauterbach et al. presented a finite element solution for Saint-Venant bending in ref. 5. Erdogan and Arin[6] considered cracked sandwich plates and performed a mathematical evaluation of the stress intensity factors. In ref. 7, a study of the effect of thickness, stiffness, and the mass of the facings on the wave propagation and vibrations in an elastic symmetrical sandwich plate was carried out. In ref. 8, the crack propagation in Lexan (PCBA) and Plexiglas (PMMA) sandwich plates was studied by using the method of dynamic caustics together with high-speed photography.

In the present work, the laminates used are made of isotropic plastic materials [Lexan (PCBA) and Plexiglas (PMMA)], forming symmetrical laminates. If multiple isotropic layers of various thicknesses are arranged symmetrically about a middle surface from both a geometric and a material property standpoint, the resulting laminate does not exhibit coupling between bending and extension. However, many physical applications of laminated composites require nonsymmetrical laminates to achieve design requirements.

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THEORETICAL ANALYSIS

A composite laminate is made of two or more layers bonded together to act as a whole structural element.

The stiffness of such a material depends on the properties of the layers.

Classical lamination theory contains a collection of stress. By the use of this theory from the initial basic building block, the lamina, we can proceed to the final element, which is a structural laminate.

The material we deal is a laminate made of 3, 4, and 5 layers of isotropic material.

In plane stress conditions, in xyz axis system, the stress–strain relationships are given.[9]

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & 0 \\ \mathbf{Q}_{12} & \mathbf{Q}_{22} & 0 \\ 0 & 0 & \mathbf{Q}_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{1}$$

The elements Q_{ij} of the stiffness matrix are related with the material properties as follows

$$\mathbf{Q}_{11} = \frac{E}{1-\nu^2} = \mathbf{Q}_{22} \quad \mathbf{Q}_{12} = \frac{\nu E}{1-\nu^2} \quad \mathbf{Q}_{66} = \frac{E}{2(1+\nu)} = G \tag{2}$$

Equation (1) can be thought of a stress–strain relationship for the kth layer of a multi-layered laminate. Thus, it can be written as

$$\{\sigma\}_k = [\mathbf{Q}]_k\{\varepsilon\}_k \tag{3}$$

By substitution of the strain variation through the thickness in this relationship, the stresses in the kth layer can be expressed in terms of the laminate middle surface strains and curvatures as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \mathbf{Q}_{11} & \mathbf{Q}_{12} & 0 \\ \mathbf{Q}_{12} & \mathbf{Q}_{22} & 0 \\ 0 & 0 & \mathbf{Q}_{66} \end{bmatrix}_k \left(\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \right) \tag{4}$$

where $\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0$ and $\kappa_x, \kappa_y, \kappa_{xy}$ are the middle surface strains and curvatures, respectively, and z is the ordinate through the thickness of the laminate.

The resultant forces and moments acting on a laminate are obtained by integration of the stresses in each layer through the laminate, for example

$$N_x = \int_{-t/2}^{t/2} \sigma_x \, dz \qquad M_x = \int_{-t/2}^{t/2} \sigma_x z \, dz \tag{5}$$

where t is the laminate thickness.

The entire force and moment resultants for an N-layered laminate is defined as

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k dz \tag{6}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-t/2}^{t/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z \, dz = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k z \, dz \tag{7}$$

where $\{N\}$ and $\{M\}$ are force per width and moment per width, respectively.

When the lamina stress–strain relations, eq. (4), are substituted and after some algebra, the following relationships are obtained.

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [\mathbf{A}_{ij}] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [\mathbf{B}_{ij}] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{8}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = [\mathbf{B}_{ij}] \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + [\mathbf{D}_{ij}] \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{9}$$

where the matrices A_{ij}, B_{ij}, D_{ij} are given as

$$\mathbf{A}_{ij} = \sum_{k=1}^{N}(\mathbf{Q}_{ij})_k(z_k - z_{k-1}) \quad \mathbf{B}_{ij} = \frac{1}{2}\sum_{k=1}^{N}(\mathbf{Q}_{ij})_k(z_k^2 - z_{k-1}^2)$$

$$\mathbf{D}_{ij} = \frac{1}{3}\sum_{k=1}^{N}(\mathbf{Q}_{ij})_k(z_k^3 - z_{k-1}^3) \tag{10}$$

In eq. (10), the A_{ij} are extensional stiffness, the $\mathbf{B}_{ij}$ are called coupling stiffness, and the $\mathbf{D}_{ij}$ are called bending stiffnesses. The presence of the $\mathbf{B}_{ij}$ implies coupling between bending and extension of a laminate. The above eqs. (8)–(9) can be written in a contracted form:

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B} & \mathbf{D} \end{bmatrix} \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \text{ or } \begin{Bmatrix} N \\ M \end{Bmatrix} = [\mathbf{K}] \begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} \tag{11}$$

From this relationship, the middle surface and curvatures can be obtained by the inversion of the matrix $[\mathbf{K}]$, i.e.

$$\begin{Bmatrix} \varepsilon^0 \\ \kappa \end{Bmatrix} = [\mathbf{F}] \begin{Bmatrix} N \\ M \end{Bmatrix}$$

where

$$[\mathbf{F}] = [\mathbf{K}]^{-1}$$

On the other hand, the elastic constants in the laminate plane E_x, E_y, ν_{xy}, and G_{xy} can be determined by the classical theory of laminated plates. Thus, Elastic modulus, shear modulus, and Poisson ratio are evaluated by the aid of models that assume uniform stress through the thickness of the laminate.

$$\frac{1}{E_x} = \mathbf{A}'_{11} \quad \frac{1}{E_y} = \mathbf{A}'_{22} \quad \nu_{xy} = \frac{-\mathbf{A}'_{12}}{\mathbf{A}'_{11}} \quad \frac{1}{G_{xy}} = \mathbf{A}'_{66} \quad (12)$$

where $\mathbf{A}'_{ij} = \mathbf{A}_{ij}^{-1}$ and $\mathbf{A}_{ij} = \frac{1}{t}[\sum_{k-1}^{N}(\mathbf{Q}_{ij})_k t_k]$.

Here t_k is the thickness of kth layer and t the thickness of the whole laminate.

EXPERIMENTAL

The specimens were made of 3–5 layers in symmetrical combination of Lexan (PCBA) and Plexiglas (PMMA) layers having 2×10^{-3} m thickness each, thus providing the form of a symmetrical laminate [see Fig. 1(a)]. For the bonding of the layers, special glue (trichloroethylene–dicloromethane 2/1) was used. To measure the strains, longitudinal and transversal strain gauges (KYOWA type with gauge length 2 mm) and Huggenberger extensometer were used.

Tensile experiments were carried out to measure mechanical properties, using dogbone specimens with
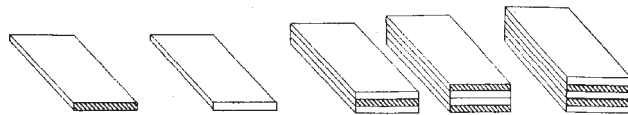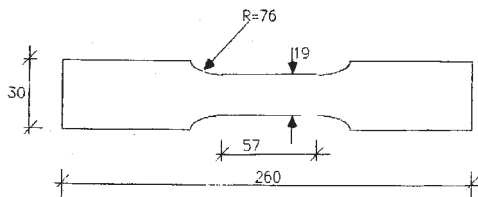

(a)
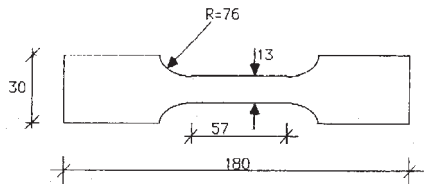
(b)

Figure 1 (a) Schematic representation of the material used. (b) Schematic representation of the specimens used for the experiments.

Material	E (GPa)	ν	σ^{ult} (MPa)
Plexiglas	3.19	0.33	61.2
Lexan	2.45	0.34	48.8

total thickness varying from 6×10^{-3} m to 10×10^{-3} m according to ASTM D638. The width of the specimens was 3×10^{-3} m near the grips and varied from 12×10^{-3} m to 19×10^{-3} m at the mid-length, whereas the total length varied from 180×10^{-3} m to 260×10^{-3} m [see Fig. 1(b)].

Special care was taken during the bonding of the various layers.

The values of the elastic modulus, E, Poisson ratio, ν, and ultimate stress, σ_{ult}, for the materials used are given in Table I.

THEORETICAL CALCULATIONS AND RESULTS

Stresses and strains

The elements of the stiffness matrix $\mathbf{Q}_{ij}$ can be calculated for each material from eq. (2) using the values of E and ν for Lexan and Plexiglas given in Table I. Thus

$$[\mathbf{Q}_{ij}]_L = \begin{bmatrix} 27702.40 & 9418.82 & 0 \\ 9418.82 & 27702.40 & 0 \\ 0 & 0 & 9141.79 \end{bmatrix} \times 10^5 \left(\frac{N}{m^2}\right) \quad (13a)$$

$$[\mathbf{Q}_{ij}]_P = \begin{bmatrix} 35798.45 & 11813.49 & 0 \\ 11813.49 & 35798.45 & 0 \\ 0 & 0 & 11992.48 \end{bmatrix} \times 10^5 \left(\frac{N}{m^2}\right) \quad (13b)$$

Now, for the first series, laminate (B1), with three layers, from eq. (10)

$$\mathbf{A}_{ij} = \{2[\mathbf{Q}_{ij}]_L + [\mathbf{Q}_{ij}]_P\}h \left(\frac{N}{m}\right) \quad (14)$$

$$\mathbf{B}_{ij} = 0 \ (N) \quad (15)$$

$$\mathbf{D}_{ij} = \{[\mathbf{Q}_{ij}]_P + 26[\mathbf{Q}_{ij}]_L\}\frac{h^3}{12}(N \ m) \quad (16)$$

where $h = t_L = t_P = 2 \times 10^{-3}m$ denotes the thickness of each layer.

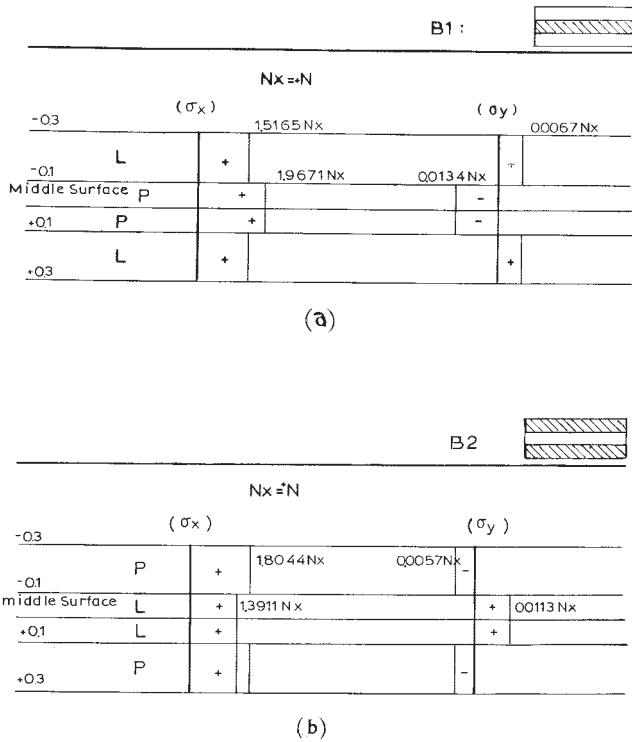The $[\mathbf{K}]$ and $[\mathbf{F}]$ matrices given in eqs. (11) and (12) are found as

$$[\mathbf{K}] = \begin{bmatrix} 18240.65 & 6130.23 & 0 & & & \\ 6130.23 & 18240.65 & 0 & & sym. & \\ 0 & 0 & 6055.21 & & & \\ 0 & 0 & 0 & 529.24 & 179.69 & 0 \\ 0 & 0 & 0 & 179.69 & 529.24 & 0 \\ 0 & 0 & 0 & 0 & 0 & 174.78 \end{bmatrix} \times 10^5 \qquad (17)$$

and

$$\mathbf{F} = \begin{bmatrix} 6 & -2 & 0 & & & \\ -2 & 6 & 0 & sym. & & \\ 0 & 0 & 17 & & & \\ 0 & 0 & 0 & 214 & -73 & 0 \\ 0 & 0 & 0 & -73 & 214 & 0 \\ 0 & 0 & 0 & 0 & 0 & 572 \end{bmatrix} \qquad (18)$$

For the second series, laminate (B2), with three layers, by the same manner we can find

$$\mathbf{A}_{ij} = \{2[\mathbf{Q}_{ij}]_P + [\mathbf{Q}_{ij}]_L\}h \left(\frac{N}{m}\right) \qquad (19)$$

$$\mathbf{B}_{ij} = 0 \ (N) \qquad (20)$$

$$\mathbf{D}_{ij} = \{[\mathbf{Q}_{ij}]_L + 26[\mathbf{Q}_{ij}]_P\}\frac{h^3}{12} \ (\text{N m}) \qquad (21)$$

$$[\mathbf{K}] = \begin{bmatrix} 19859.86 & 6609.16 & 0 & & sym. & \\ 6609.16 & 19859.86 & 0 & & & \\ 0 & 0 & 6625.35 & & & \\ 0 & 0 & 0 & 670.92 & 221.60 & 0 \\ 0 & 0 & 0 & 221.60 & 670.92 & 0 \\ 0 & 0 & 0 & 0 & 0 & 224.66 \end{bmatrix} \times 10^5 \qquad (22)$$

and

$$[\mathbf{F}] = \begin{bmatrix} 6 & -2 & 0 & & & \\ -2 & 6 & 0 & & sym. & \\ 0 & 0 & 15 & & & \\ 0 & 0 & 0 & 167 & -55 & 0 \\ 0 & 0 & 0 & -55 & 167 & 0 \\ 0 & 0 & 0 & 0 & 0 & 445 \end{bmatrix} \qquad (23)$$

If we compare the matrix $[\mathbf{K}]$ of this laminate with the previous one (B1), we observe that there is a slight increase in the terms, which is due to the different stacking sequence of the Lexan and Plexiglas layers.

For the third series, laminate (C1), with four layers, from eq. (10)

$$\mathbf{A}_{ij} = \{[\mathbf{Q}_{ij}]_L + [\mathbf{Q}_{ij}]_P\}2h \ (\text{N/m}) \qquad (24)$$

$$\mathbf{B} = 0 \ (\text{N}) \qquad (25)$$

$$\mathbf{D}_{ij} = \{7[\mathbf{Q}_{ij}]_P + [\mathbf{Q}_{ij}]_L\}\frac{2h^3}{3} \ (\text{N m}) \qquad (26)$$

$$[\mathbf{K}] = \begin{bmatrix} 25400.34 & 8492.92 & 0 & & & \\ 8492.92 & 25400.34 & 0 & & sym. & \\ 0 & 0 & 8453.71 & & & \\ 0 & 0 & 0 & 1474.95 & 488.20 & 0 \\ 0 & 0 & 0 & 488.20 & 1474.95 & 0 \\ 0 & 0 & 0 & 0 & 0 & 493.37 \end{bmatrix} \times 10^5 \qquad (27)$$

and

$$[\mathbf{F}] = \begin{bmatrix} 4 & -1 & 0 & & & \\ -1 & 4 & 0 & sym. & & \\ 0 & 0 & 12 & & & \\ 0 & 0 & 0 & 76 & -25 & 0 \\ 0 & 0 & 0 & -25 & 76 & 0 \\ 0 & 0 & 0 & 0 & 0 & 203 \end{bmatrix} \qquad (28)$$

If we compare the matrix $[\mathbf{K}]$ with that of laminates (B1) and (B2), we observe that there is a considerable increase in all terms something expected, since the thickness of the laminate increased.

For the fourth series, laminate (E1), with five layers, from eq. (10)

$$\mathbf{A}_{ij} = \left\{\frac{3}{2}[\mathbf{Q}_{ij}]_L + [\mathbf{Q}_{ij}]_P\right\}2h \ (\text{N/m}) \qquad (29)$$

$$\mathbf{B}_{ij} = 0 \ (\mathrm{N}) \tag{30}$$

$$\mathbf{D}_{ij} = \left\{ [\mathbf{Q}_{ij}]_P + \frac{99}{26}[\mathbf{Q}_{ij}]_L \right\} \frac{13h^3}{6} \ (\mathrm{N\ m}) \tag{31}$$

$$[\mathbf{K}] = \begin{bmatrix} 30940.82 & 10376.69 & 0 & & & \\ 10376.69 & 30940.82 & 0 & & sym. & \\ 0 & 0 & 10287.07 & & & \\ 0 & 0 & 0 & 2444.16 & 824.82 & 0 \\ 0 & 0 & 0 & 824.82 & 2444.16 & 0 \\ 0 & 0 & 0 & 0 & 0 & 80967 \end{bmatrix} \times 10^5 \tag{32}$$

and

$$[\mathbf{F}] = \begin{bmatrix} 4 & -1 & 0 & & & \\ -1 & 4 & 0 & & sym. & \\ 0 & 0 & 9 & & & \\ 0 & 0 & 0 & 46 & -16 & 0 \\ 0 & 0 & 0 & -16 & 46 & 0 \\ 0 & 0 & 0 & 0 & 0 & 124 \end{bmatrix} \tag{33}$$

Again, the terms of matrix $[\mathbf{K}]$ of this laminate (E1) show an increase when compared with the terms of laminate (C1) due to the increase of thickness.

$$[\mathbf{K}] = \begin{bmatrix} 32560.03 & 10833.02 & 0 & & & \\ 10855.62 & 32560.03 & 0 & & sym. & \\ 0 & 0 & 10852.20 & & & \\ 0 & 0 & 0 & 2837.41 & 941.14 & 0 \\ 0 & 0 & 0 & 941.16 & 2837.41 & 0 \\ 0 & 0 & 0 & 0 & 0 & 948.14 \end{bmatrix} \times 10^5 \tag{37}$$

and

$$[\mathbf{F}] = \begin{bmatrix} 3 & -1 & 0 & & & \\ -1 & 3 & 0 & sym. & & \\ 0 & 0 & 9 & & & \\ 0 & 0 & 0 & 40 & -13 & 0 \\ 0 & 0 & 0 & -13 & 40 & 0 \\ 0 & 0 & 0 & 0 & 0 & 105 \end{bmatrix} \tag{38}$$

Now, if we compare the matrix $[\mathbf{K}]$ of this laminate (E2) with the previous one of Lam. (E1), we observe again that there is a slight increase in the terms due to the different stacking sequence of the Lexan and Plexiglas layers.

To proceed to the calculation of the stresses of the material, let us take into account eq. (12).

For the first series, laminate (B1), by the aid of eq. (18), we obtain

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} 6 & -2 & 0 & & & \\ -2 & 6 & 0 & sym. & & \\ 0 & 0 & 17 & & & \\ 0 & 0 & 0 & 214 & -73 & 0 \\ 0 & 0 & 0 & -73 & 214 & 0 \\ 0 & 0 & 0 & 0 & 0 & 572 \end{bmatrix} \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x \\ M_y \\ M_{xy} \end{Bmatrix} \tag{39}$$

The strains are calculated by taking under consideration eqs. (3) and (4) as

For the fifth series, laminate (E2), again with five layers, from eq. (10)

$$\mathbf{A}_{ij} = \left\{ \frac{3}{2}[\mathbf{Q}_{ij}]_P + [\mathbf{Q}_{ij}]_L \right\} 2h \ (\mathrm{N/m}) \tag{34}$$

$$\mathbf{B}_{ij} = 0 \ (\mathrm{N}) \tag{35}$$

$$\mathbf{D}_{ij} = \left\{ [\mathbf{Q}_{ij}]_L + \frac{99}{26}[\mathbf{Q}_{ij}]_P \right\} \frac{13h^3}{6} \ (\mathrm{N\ m}) \tag{36}$$

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} \tag{40}$$

For uniaxial tension $N_x = N$, $N_y = N_{xy} = M_x = M_y = M_{xy} = 0$. Thus $\varepsilon_x = 6 \times 10^{-5} N_x$, $\varepsilon_y = -2 \times 10^{-5} N_x$, and $\gamma_{xy} = \kappa_x = \kappa_y = \kappa_{xy} = 0$. By substituting these values in eq. (3), the stresses in each layer can be found as

In Lexan $\sigma_x = 1.5165 N_x$, $\sigma_y = 0.0067 N_x$, $\tau_{xy} = 0$
In Plexiglas: $\sigma_x = 1.9671 N_x$, $\sigma_y = -0.0134 N_x$, $\tau_{xy} = 0$

It is obvious that while the stress σ_x is positive in both layers, negative σ_y develops in Plexiglas. This is due to the fact that Lexan tends to deform transversally more than Plexiglas. But, since this is hindered, it results to the creation of σ_y.

The variations of stresses is illustrated in Figure 2(a). From this figure, it can be observed that σ_y fulfils the equilibrium condition since $2 \times 0.0067 N_x - 0.0134 N_x = 0$.

For laminate (B2), by similar procedure we obtain the following:

(a)

(b)

Figure 2　Variation of laminate stresses σ_x and σ_y for laminates B1 and B2.

In Lexan: $\sigma_x = 1.391N_x$, $\sigma_y = 0.0113N_x$, $\tau_{xy} = 0$
In Plexiglas: $\sigma_x = 1.8044N_x$, $\sigma_y = -0.0057N_x$, $\tau_{xy} = 0$

The variation is illustrated in Figure 2(b).
For laminate (C1) in a similar way we obtain

In Lexan: $\sigma_x = 1.0883N_x$, $\sigma_y = 0.0069N_x$, $\tau_{xy} = 0$
In Plexiglas: $\sigma_x = 1.4117N_x$, $\sigma_y = -0.0069N_x$, $\tau_{xy} = 0$

The variation of stresses is illustrated in Figure 3.
For laminate (E1) similarly we obtain:

In Lexan: $\sigma_x = 0.8938N_x$, $\sigma_y = -0.0046N_x$, $\tau_{xy} = 0$
In Plexiglas: $\sigma_x = 1.1593N_x$, $\sigma_y = -0.0070N_x$, $\tau_{xy} = 0$

The variation is illustrated in Figure 4.
Finally for laminate (E2), we obtain

In Lexan: $\sigma_x = 0.8487N_x$, $\sigma_y = -0.0063N_x$, $\tau_{xy} = 0$
In Plexiglas: $\sigma_x = 1.1009N_x$, $\sigma_y = -0.0042N_x$, $\tau_{xy} = 0$

The variation is illustrated in Figure 5.
Next, the strains for the case of uniaxial tension will be calculated. To make comparison in each case, the strains for equivalent Lexan and Plexiglas material also are given.
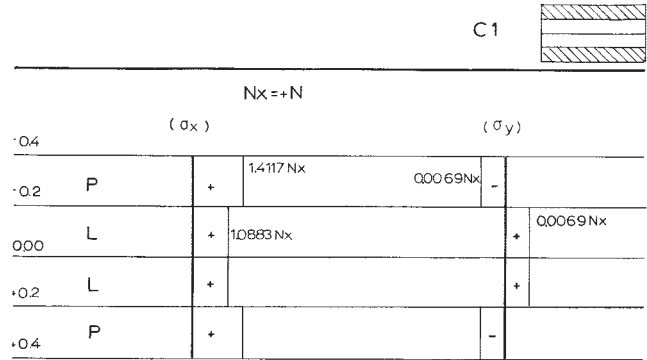
Figure 3　Variation of laminate stresses σ_x and σ_y for lam. C1.

For laminate (B1) and for N_x, force per unit width in $\left(\dfrac{N}{m}\right)$ or $\left(\dfrac{kp}{cm}\right)$, if we use Lexan with total thickness $t = 3h = 3 \times 0.2 = 0.6$ cm, then

$$\varepsilon_x = \frac{\sigma_x}{E_L} = \frac{N_x/b\,t}{E_L} = \frac{N_x}{3h \times 24500} = 6.8 \times 10^{-5}N_x$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.34\varepsilon_x = -2.3 \times 10^{-5}N_x$$

If we use Plexiglas with $t = 3h = 3 \times 0.2 = 0.6$ cm, then the strain distribution is

$$\varepsilon_x = \frac{\sigma_x}{E_p} = \frac{N_x/bt}{E_p} = \frac{N_x}{3h \times 31900} = 5.2 \times 10^{-5},$$

$$\varepsilon_y = -\nu\varepsilon_x = -0.33\varepsilon_x = -1.7 \times 10^{-5}N_x.$$

By the aid of eqs. (12) and (40), we obtain

In Lexan: $\varepsilon_x = 6.8 \times 10^{-5}N_x$, $\varepsilon_y = -2.3 \times 10^{-5}N_x$, $\gamma_{xy} = 0$
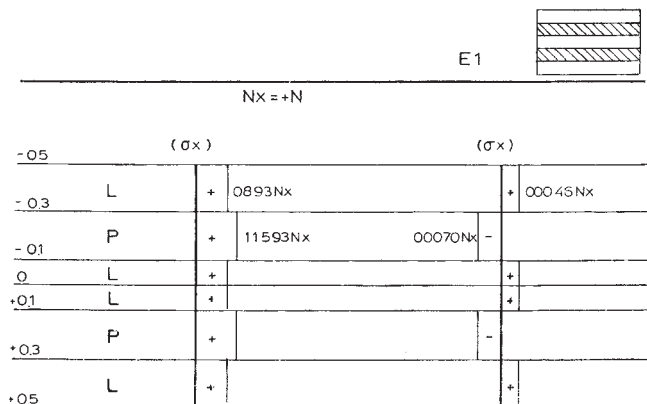
Figure 4　Variation of laminate stresses σ_x and σ_y for lam. E1.
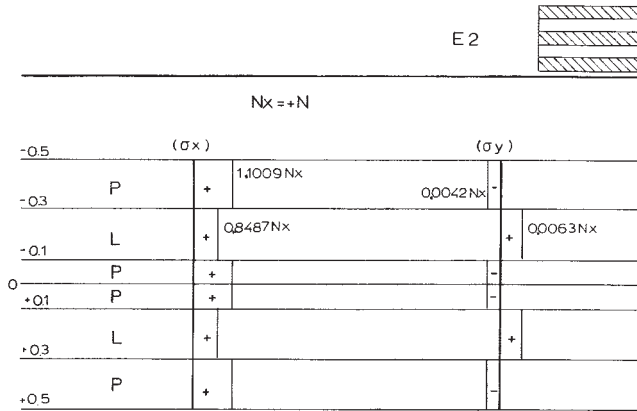
Figure 5 Variation of laminate stresses σ_x and σ_y for lam. E2.

Figure 7 Variation of laminate strains ε_x and ε_y for lam. B2.

In Plexiglas: $\varepsilon_x = 5.2 \times 10^{-5}N_x$, $\varepsilon_y = -1.7 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = 6.0 \times 10^{-5}N_x$, $\varepsilon_y = -2.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$
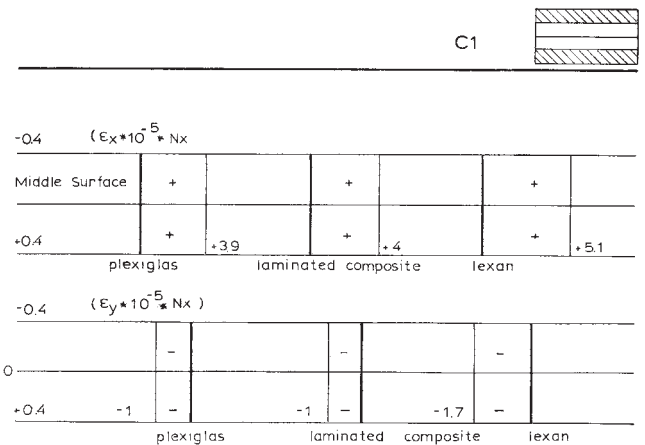
The variation of these strains is illustrated in Figure 6.

For laminate (B2) through a similar procedure we obtain:

In Lexan: $\varepsilon_x = 6.8 \times 10^{-5}N_x$, $\varepsilon_y = -2.3 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 5.2 \times 10^{-5}N_x$, $\varepsilon_y = -1.7 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = 6.0 \times 10^{-5}N_x$, $\varepsilon_y = -2.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 7. It can be observed that the strains are the same as in the previous case, although the stacking sequence is different.

For laminate (C1) with $t = 4h = 4 \times 0.2 = 0.8$ cm we obtain

In Lexan: $\varepsilon_x = 5.1 \times 10^{-5}N_x$, $\varepsilon_y = -1.7 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 3.9 \times 10^{-5}N_x$, $\varepsilon_y = -1.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = 4 \times 10^{-5}N_x$, $\varepsilon_y = -1.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 8.

For laminate (E1) with $t = 5h = 5 \times 0.2 = 1$ cm, we obtain

In Lexan: $\varepsilon_x = 4.1 \times 10^{-5}N_x$, $\varepsilon_y = -1.4 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 3.1 \times 10^{-5}N_x$, $\varepsilon_y = -1.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In the Laminate: $\varepsilon_x = 4.0 \times 10^{-5}N_x$, $\varepsilon_y = -1.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 9.

Finally, for laminate (E2), we obtain

In Lexan: $\varepsilon_x = 4.1 \times 10^{-5}N_x$, $\varepsilon_y = -1.4 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

In Plexiglas: $\varepsilon_x = 3.1 \times 10^{-5}N_x$, $\varepsilon_y = -1.0 \times 10^{-5}N_x$, $\gamma_{xy} = 0$

Figure 6 Variation of laminate strains ε_x and ε_y for lam. B1.

Figure 8 Variation of laminate strains ε_x and ε_y for lam. C1.
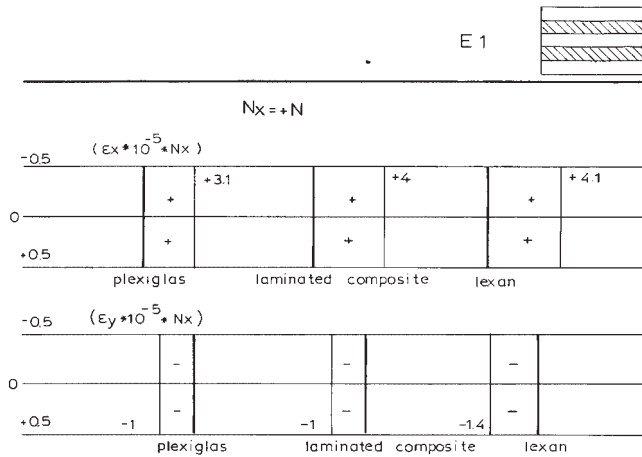
Figure 9 Variation of laminate strains ε_x and ε_y for lam. E1.

In the Laminate: $\varepsilon_x = 3.0 \times 10^{-5}N_x$, $\varepsilon y = -1.0 \times 10{-}5Nx$, $\gamma_{xy} = 0$

The variation is illustrated in Figure 10. It can be easily observed that the strain ε_x is different in this case when compared with the previous one, whereas the strain ε_y is the same although the number of layers and the total thicknesses are equal but the stacking sequence is different.

Elastic constants

Let us now calculate the elastic constants of the different laminates used. In this analysis, the laminate is considered as consisted of a layer of a homogeneous material. The calculations will be carried out through eqs. (12) and through the approximate formulae of the rule of mixtures.

$$E_c = E_L U_L + E_p U_p \qquad (41)$$
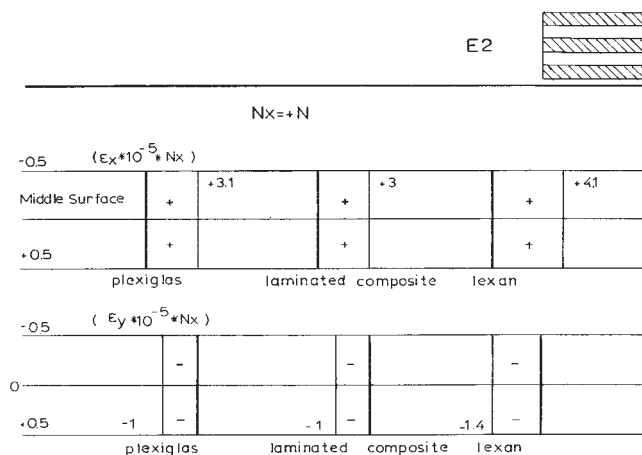
Figure 10 Variation of laminate strains ε_x and ε_y for lam. E2.

TABLE II
Theoretical Values of the Laminate Elastic Modulus

Laminate	E_c (GPa)	$(E_c)_{approx}$ (GPa)
B1	2.697	2.696
B2	2.943	2.946
C1	2.820	2.820
E1	2.746	2.746
E2	2.894	2.894

$$\nu_c = \nu_L U_L + \nu_p U_f \qquad (42)$$

where E_L, ν_L, and U_L denote the elastic modulus, Poisson ratio, and volume fraction of the Lexan and E_p, ν_p, and U_p are those of the Plexiglas, respectively. The volume fractions are given as the ratios of the volume of each material to the total volume of the laminate and with $U_L + U_p = 1$. Using the values given in Table I for E and ν and the values of $[\mathbf{K}]$ matrix for $\mathbf{A}_{ij}$, $\mathbf{B}_{ij}$ and $\mathbf{D}_{ij}$ given in eqs. (17), (22), (27), (32), and (37) by the aid of eqs. (12) and (2), the values of the laminate elastic modulus and Poisson ratio can be obtained. These values appear in Table II and Table III, respectively. It can be observed that there is a very good coincidence between the values of Poisson ratio calculated by the laminate theory given in eqs. (12) and those calculated by the approximate theory of the rule of mixtures given in eq. (42). Also, it can be said that the stacking sequence of the laminate does not influence the Poisson ratio, whereas it influences slightly the elastic modulus the maximum value of which is for Lam. B2 and the minimum for Lam. B1. The ranking for the elastic modulus for the various laminates is B1, E1, C1, E2, and B2, whereas for the Poisson ratio is B2, E2, C1, E1, and B1, which is the opposite. Thus, it can be said that the laminates with the least number of layers constitute the maximum and the minimum for the elastic constants.

Determination of the ultimate load carrying capacity

It is required to determine the ultimate load carrying capacity of a laminate, defined as one consisting of two or more dissimilar materials, under a tensile load P. The laminate is composed of 3, 4, or 5 layers of

TABLE III
Theoretical Values of the Laminate Poisson Ratio

Laminate	ν_c	$(\nu_c)_{app}$
B1	0.336	0.337
B2	0.333	0.333
C1	0.334	0.335
E1	0.335	0.336
E2	0.333	0.334

isotropic material (Lexan/Plexiglas) with thickness 2 mm for each layer as mentioned previously.

Since strains in all layers at a particular cross section are equal

$$\varepsilon_L = \varepsilon_P \to \frac{\sigma_L}{E_L} = \frac{\sigma_P}{E_P} \qquad (43)$$

which yields

$$\sigma_L = \frac{E_L}{E_P}\sigma_P \text{ or } \sigma_P = \frac{E_P}{E_L}\sigma_L \qquad (44a,b)$$

From these relationships by using the values of Table I, we have

(a) If $\sigma_c^{\text{ult.}} = \sigma_P^{\text{ult.}} = 61.2 \text{ MPa} \to \sigma_L = 47 \text{ MPa} < \sigma_L^{\text{ult.}}$

$$(45)$$

(b) If $\sigma_c^{\text{ult.}} = \sigma_L^{ult.} = 48.8 \text{ MPa} \to \sigma_P = 63.54 \text{ MPa} > \sigma_P^{\text{ult.}}$

$$(46)$$

The subscripts C,L,P denote the composite, the Lexan, and the Plexiglas, respectively. The first relationship states that when the ultimate stress for Plexiglas is reached, the stress value in Lexan is less than its ultimate, whereas the second relationship states that when the ultimate stress for Lexan is reached, the stress value in Plexiglas has been exceeded.

Therefore, the criteria of failure of the laminate is Plexiglas and the mean ultimate load carrying capacity of the material is

1. For series B

$$P = \sigma_P^{\text{ult.}} \times b_P \times t_P + \sigma_L \times b_L \times t_L \times 2 \qquad (47)$$

where b_P and b_L denote the width and t_P and t_L denote the thickness of the Lexan/Plexiglas layers, which are equal to h.

The above relationship by using the values of eqs. (44)–(46) given that the thickness of each layer was 2 mm and the nominal width varied from 12 to 19 mm depending on each series (in this one is 12.27 mm) yields

$$P = (61.2 \times 10^6 \times 12.27 \times 10^{-3} \times 2 \times 10^{-3} + 47$$
$$\times 10^6 \times 12.27 \times 10^{-3} \times 2 \times 10^{-3} \times 2) = 3808.6 \text{ N}$$

A check should be made on the ultimate load on the laminate after the failure of Plexiglas. In this case, the ultimate load that the material is able to support is

$$P' = 48.8 \times 10^6 \times 12.27 \times 10^{-3} \times 2$$
$$\times 10^{-3} \times 2 = 2395 \text{ N}$$

2. For series C

$$P = \sigma_P^{\text{ult.}} \times b_P \times t_P \times 2 + \sigma_L \times b_L \times t_L \times 2 \qquad (48)$$

The above relationship by using the values of eqs. (13a) and (13b) and given that the thickness of each layer was 2 mm and the mean width 18.07 mm yields

$$P = (61.2 \times 10^6 \times 18.07 \times 10^{-3} \times 2 \times 10^{-3} \times 2 + 47$$
$$\times 10^6 \times 18.07 \times 10^{-3} \times 2 \times 10 \times 2) = 7820.7 \text{ N}$$

In this case, the ultimate load that the material is able to support is

$$P' = 48.8 \times 10^6 \times 18.07 \times 10^{-3} \times 2$$
$$\times 10^{-3} \times 2 = 3527.3 \text{ N}$$

3. For series E

$$P = \sigma_P^{\text{ult.}} \times b_P \times t_P \times 2 + \sigma_L \times b_L \times t_L \times 3 \qquad (49)$$

Similarly, the above relationship given that the thickness of each layer was 2 mm and the mean width 18.04 mm yields

$$P = (61.2 \times 10^6 \times 18.04 \times 10^{-3} \times 2 \times 10^{-3} \times 2 + 47$$
$$\times 10^6 \times 18.04 \times 10^{-3} \times 2 \times 10^{-3} \times 3) = 9503.4 \text{ N}$$

Again, in this case, the ultimate load that the materials able to support is

$$P' = 48.8 \times 10^6 \times 18.04 \times 10^{-3} \times 2 \times 10^{-3} \times 3$$
$$= 5282.1 \text{ N}$$

RESULTS

Figure 11 illustrates the initial part of the stress–strain diagram for Lexan and Plexiglas as derived from tensile experiments by using mechanical gauges (Hüggenberger) for the determination of the longitudinal strain. From these diagrams the elastic moduli of Lexan and Plexiglas were evaluated as 2.45 GPa and 3.19 GPa, respectively. Both diagrams show a strong linear behavior for the variation of the stress versus strain for the two materials. It can be observed that Lexan is a ductile material and its cross section decreases until the rupture of the material. Its failure almost coincides with its yielding and it is a material that it can be "trusted" when working in large strains. On the contrary, Plexiglas looks like a brittle material but its strength is better than Lexan. As a conclusion it
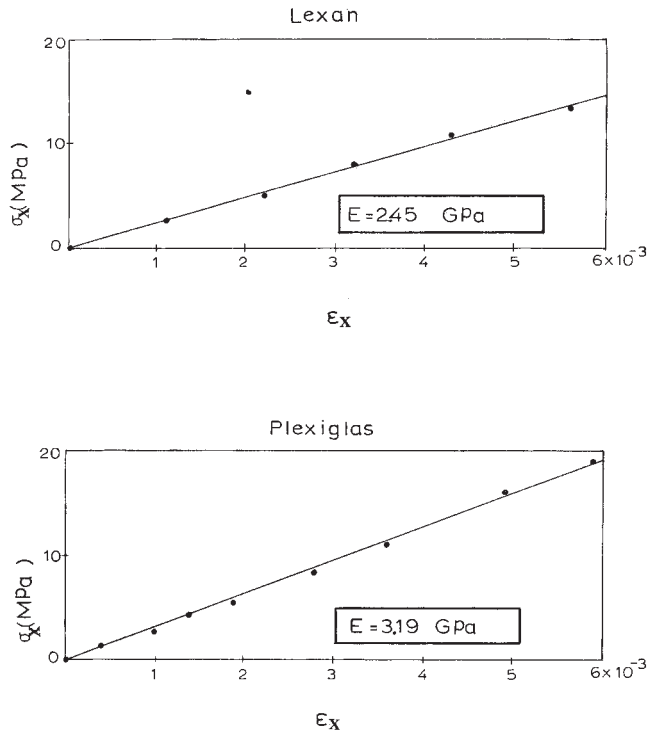
Figure 11 Variation of stress σ_x vs. strain ε_x for the Lexan and Plexiglas as derived from tensile experiments using mechanical gauges.

can be said that from the combination of these two materials and through different stacking sequences, a mean behavior can be expected. In this, Lexan normally will contribute by its ductility and Plexiglas by its strength.

Figures 12–14 present the initial part of the stress-strain diagram for the series B1, C1, and E1, respectively, as obtained from tensile experiments by using again mechanical gauges for the determination of the longitudinal strain. From these diagrams the elastic moduli for the three material were evaluated as 3.01 GPa, 3.10 GPa, and 2.87 GPa, respectively. Again,

Figure 13 Variation of stress σ_x vs. strain ε_x for the lam. C1 used as derived from tensile experiments using mechanical gauges.

in all diagrams, it can be observed that the variation of the stress versus strain shows a strong linear behavior. The variation of stress versus strain for the series B1, C1, and E1 where the percentage of Plexiglas is 1/3, 1/2, and 2/5, respectively, as obtained from tensile experiments by using electrical strain-gauges is illustrated in Figures 15–17. The initial part of the diagrams, served for the evaluation of the elastic modulus appears in Figures 15(a)–17(a).

The mean values for ultimate stress, elastic modulus, and Poisson ratio obtained experimentally are presented in Table IV.

It can be observed that Lam. B1 presents the highest strength, although the percentage of Plexiglas, which has higher strength than lexan, is the lowest of the series. On the other hand, Lam. C1 presents the highest elastic modulus that is reasonable, since this series has the highest percentage of Plexiglas and the elastic modulus of which is higher than lexan. Normally, the laminate with higher percentage of Plexiglas should have higher elastic modulus as it can be seen in Table II in the theoretical values of elastic modulus, which were evaluated as 2.70 GPa, 2.82 GPa, and 2.75 GPa for laminates B1, C1, and E1, respectively, the fact that it is not true in the experimental results where Lam. B1
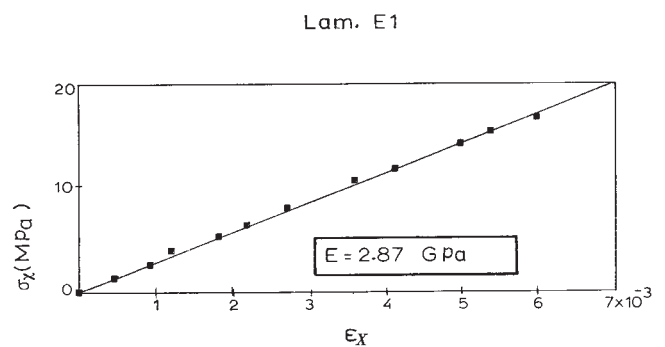

Figure 12 Variation of stress σ_x vs. strain ε_x for the lam. B1 used as derived from tensile experiments using mechanical gauges.

Figure 14 Variation of stress σ_x vs. strain ε_x for the lam. E1 used as derived from tensile experiments using mechanical gauges.
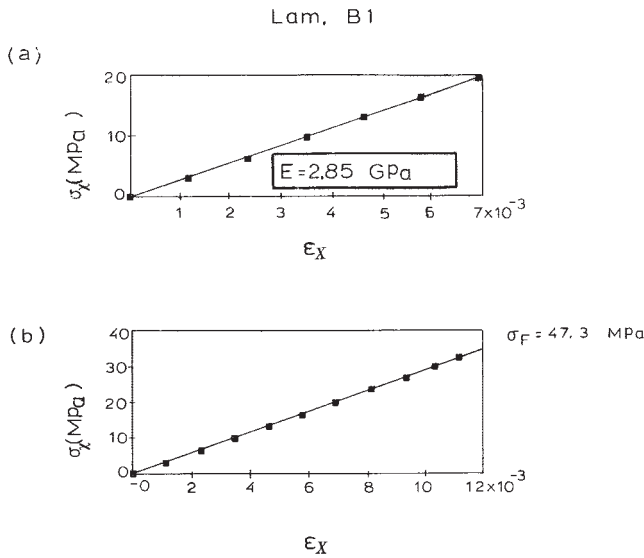
Lam. B1

Figure 15 Variation of stress σ_x vs. strain ε_x for the lam. B1 used as derived from tensile experiments using strain-gauges (a) initial part and (b) entire curve.

has the highest elastic modulus from the measurements received through strain-gauges. On the contrary, from the measurements received through mechanical strain-gauges, the elastic modulus of Lam. C1 is the highest that is in accordance with the theoretical values. The discrepancy can be attributed to the difference between the two types of gauges.

It is worth mentioning that the elastic moduli obtained through mechanical gauges are higher than those obtained through electrical strain-gauges, which
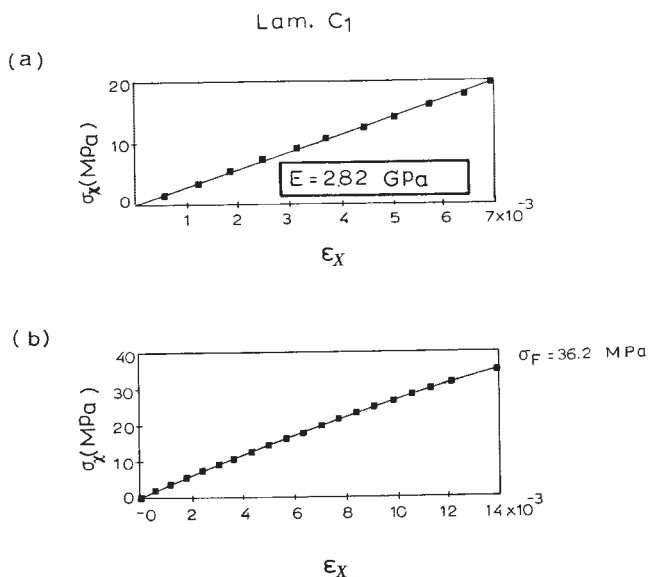
Lam. C1

Figure 16 Variation of stress σ_x vs. strain ε_x for the lam. C1 used as derived from tensile experiments using strain-gauges (a) initial part and (b) entire curve.

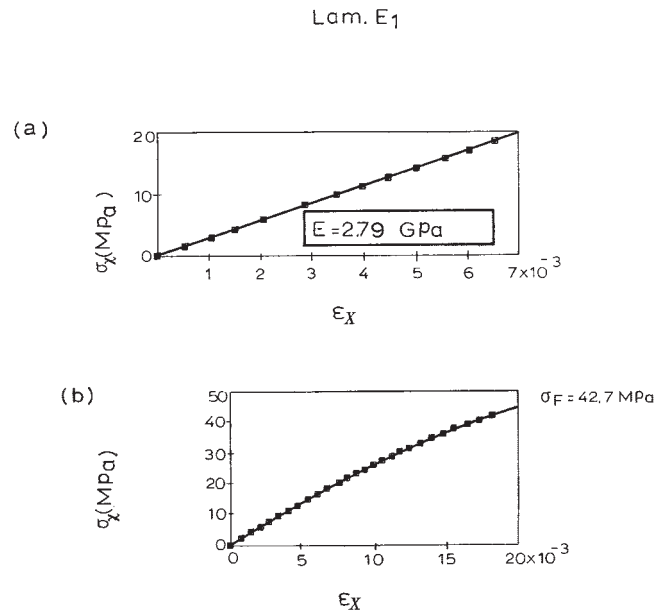Lam. E1

Figure 17 Variation of stress σ_x vs. strain ε_x for the lam. E1 used as derived from tensile experiments using strain-gauges (a) initial part and (b) entire curve.

in turn yield elastic moduli higher than the theoretical results.

The experimental Poisson ratio, ν_c, of the laminates was determined by the ratio of the transversal strain to the longitudinal strain. The mean values are presented in Table IV. It can be observed that Lam. B1 has the highest Poisson ratio among the laminates investigated and that these values present similar behavior as the ultimate stress. On the other hand, it can be said that there is a discrepancy between the experimental results obtained and the theoretical values of Table III for all types of laminates, and that in all cases, experimental values are superior to the theoretical ones.

As to the failure of the laminates, a general observation concerning the fracture mechanism can be stated: when the applied load increases, the bonding at the interface of the layers seems to weaken and failure occurs in lines lying at planes perpendicular to the loading direction. The phenomenon starts from the neck of the specimen and moves up and down towards the grips. It can observed that this phenomenon progresses even in the grips. This continues up to whitening covers the specimen. The fracture surface is almost plane and perpendicular to the specimen axis, which means that the failure occurred only from normal stresses. After the fracture of the specimens, an effort was made to separate the layers of the materials from each other, something that was impossible. The layers continued to be strongly bonded, the fact that led to the conclusion that the cracks started in a material

TABLE IV
Experimental Values of Elastic Modulus, Poisson Ratio, and Ultimate Stress of the Materials Used

| Material | Experimental modulus | | Experimental Poisson ratio ν_c | Experimental ultimate stress σ_c^{ult} (MPa) |
	E_c (GPa) (Hüggenberger)	E_c (GPa) (strain-gauge)		
Lexan	2.45		0.34	48.8
Plexiglas	3.19		0.33	61.2
Lam.B1	3.01	2.85	0.38	00.1
Lam.Cl	3.10	2.82	0.36	57.8
Lam.E1	2.87	2.79	0.37	58.0

rather than in an interface. This material probably was Plexiglas, which as a less ductile material cannot follow the large deformations of Lexan. This hypothesis is reinforced by the fact that the more was the percentage of Lexan the more "whitening" appeared before fracture and the more was the final deformation.

Finally, if we compare the ultimate load carrying capacity of the laminates under a tensile load P as derived from eqs. (44)–(46) with experimental results [see Table V], we can observe that there is a discrepancy between the "theoretical" values and the mean experimental ones and that the experimental results are in all cases larger. This means the construction of the laminates examined was good.

CONCLUSIONS

From the comparison of theoretical and experimental results of the different types of laminates we conclude

TABLE V
Experimental and "Theoretical" Ultimate Load Carrying Capacity of the Laminates Used

Material	P_{exp} (N)	P (N)
Lam.B1	4,576.7	3,808.6
Lam.El	8,460.2	7,820.7
Lam.E1	10,625.1	9,503.4

1. The appropriate combination of the layers and the position of each one in the laminate can have as result the increase of the strength.
2. Depending on the position of each layer and the combination in the laminate, an increase in the experimental values of the elastic modulus and Poisson ratio appears compared with the respective theoretical ones.
3. The Elastic modulus of all types of the laminates examined remains between the values of the pure Lexan and pure Plexiglas, i.e., $E_{\text{Lex}} < E_{\text{Lam}} < E_{\text{Pl}}$.
4. The mean experimental values for the ultimate load are greater than the "theoretical" ones but the difference is not too considerable.

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